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·基础理论研究·

The optimal harvesting policy of stage-structured predator-prey system

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Abstract: Predator-prey models play a crucial role in bioeconomics, that is the management of renewable resources. A bioeconomic model of two species with stage structure and the relation of predator-prey is established, the necessary and sufficient condition for the permanence of two species and the extinction of one species or two species is obtained. The optimal harvesting and the threshold of harvesting for the sustainable development is also obtained.

Key words: bioeconomic modelling; permanence; the optimal harvesting policy

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The competitive, cooperative and predator-prey models have been studied by many authors (see monographs [1, 2, 3, 4, 5, 6, 7]). The permanence (or strong persistence) and extinction are significant concepts of those models. However, the stage structure of species has been considered very little. In the real world, almost all animals have the stage structure of immature and mature. Recently, papers [8, 9] studied the stage structure of species, the transformation rate of mature population is proportional to the existing immature population; papers [10, 11, 12] also studied the stage-structured models, the time from birth to maturity represented by a constant time delay. Cannibalish models of various types have also been investigated (see [13, 14, 15, 16]).

The optimal management of renewable resources, which has a direct relationship to sustainable development, has been studied extensively by many authors. Economic and biological aspects of renewable resources management have been considered by Clark[17] and other authors. Predator-prey models play a crucial role in bioeconomics, that is the management of renewable resources. When practiced, the management of renewable resources has been based on the MSY, abbreviation for maximum sustainable yields. The MSY is a simple way to manage resources taking into consideration that overexploiting resources lead to a loss in productivity. Based on a biological growth model, the MSY depends upon the environmental carrying capacity K. As the population approaches the value K, the surplus production approaches zero. Therefore, the aim is to determine how much we can harvest without altering dangerously the harvested population.

We intend to consider the stage structure of two species. For the simplicity of our model, we only consider the stage structure of immature and mature of the prey species (their sizes of population are written as u_1, u_2 respectively), and not consider the stage structure of the predator species (its size of population is written as v), and two species satisfy the following assumptions:

(H_1) The birth rate of the immature prey population is proportional to the existing mature

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prey population with a proportionality constant b; for the immature prey population, the death rate and transformation rate of mature prey are proportional to the existing immature prey population with proportionality constants r_1 and b_1.

(H_2) The death rate and overcrowding rate of the mature prey population is of logistic nature, i.e. it is proportional to square of the population with a proportionality contant $r_2 > 0$. qEu_2 is the harvesting yield. q is the catachability coefficient and E is the harvesting effort.

(H_3) The growth of the predator population is of Lotka-Volterra nature. The predator population feed on the mature prey. This seems reasonable for a number of animals, where immature prey populations concealed in the mountain cave, are raised by their parents; the rate predators attack at immature prey can be ignored.

According to (H_1), (H_2) and (H_3), we can set up the following stage-structured predator-prey model

$$\begin{cases} \mathring{u}_1 = bu_2 - r_1u_1 - b_1u_1 \\ \mathring{u}_2 = b_1u_1 - r_2u_2^2 - a_1u_2v - qEu_2 \\ \mathring{v} = v(-r + a_2u_2 - b_2v) \end{cases} \quad (1)$$

where $b, b_1, b_2, r, r_1, r_2, a_1, a_2, q$ are positive constants, $\mathring{u}_i = du_i/dt$, $\mathring{v} = dv/dt$.

Let $x_1 = a_2b^{-1}u_1$, $x_2 = a_2(r_1 + b_1)^{-1}u_2$, $y = b_2(r_1 + b_1)^{-1}v$, $d\tau = (r_1 + b_1)dt$, system (1) can be turned into

$$\begin{cases} \mathring{x}_1 = x_2 - x_1 \\ \mathring{x}_2 = \beta_1x_1 - \beta_2x_2^2 - ax_2y - E_1x_2 \\ \mathring{y} = y(-d + x_2 - y) \end{cases} \quad (2)$$

where $\mathring{x}_i = dx_i/d\tau$, $\mathring{y} = dy/d\tau$, $\beta_1 = b_1b(r_1 + b_1)^{-2}$, $\beta_2 = r_2a_2^{-1}$, $a = a_1b_2^{-1}$, $d = r(r_1 + b_1)^{-1}$, $E_1 = qE(r_1 + b_1)^{-1}$.

In the next section, we shall consider the condition of permanence and extinction of system. The exploitation of the mature population is considered in section 3.

1 Permanence and extinction of system (2)

At first, we give the following notations and definitions

$$R_+^3 = \{x = (x_1, x_2, y) \in R^3, x_i \geq 0, y \geq 0\},$$
$$\text{Int}R_+^3 = \{x = (x_1, x_2, y) \in R^3, x_i > 0, y > 0\}.$$

Definition 1 System (2) is said to permanence if there are positive constants m and M such that each positive solution $x(t, x_0)$ of (2) with initial condition $x_0 \in \text{Int}R_+^3$ satisfies

$$m \leq \lim_{t\to\infty} \inf x_i(t, x_0) \leq \lim_{t\to\infty} \sup x_i(t, x_0) \leq M,$$
$$m \leq \lim_{t\to\infty} \inf y(t, x_0) \leq \lim_{t\to\infty} \sup y(t, x_0) \leq M.$$

Definition 2 (i) The prey species of system (2) is said to extinction if each positive solution $x(t, x_0)$ of system (2) with initial conditial condition $x_0 \in \text{Int}R_+^3$ satisfies

$$\lim_{t\to\infty} x_i(t, x_0) = 0, i = 1, 2.$$

(ii) The predator species of system (2) is said to extinction if each positive solution $x(t, x_0)$ of system (2) with initial condition $x_0 \in \text{Int}R_+^3$ satisfies

$$\lim_{t\to\infty} y(t, x_0) = 0.$$

Definition 3 A constant δ_0 is said to be threshold (critical situations) of the harvesting effort (or harvesting) if system (2) is permanence as the harvesting effort $E_1 < \delta_0$ (or the harvesting yield $h < \delta_0$), and at least one species of system (2) will be extinction as the harvesting effort $E_1 > \delta_0$ (or harvesting yield $h > \delta_0$).

The equilibria for our model are determined by setting $\mathring{x}_1 = \mathring{x}_2 = \mathring{y} = 0$ in system (2) and solving the resulting algebraic equations

$$\begin{cases} x_2 - x_1 = 0, \\ \beta_1x_1 - \beta_2x_2^2 - ax_2y - E_1x_2 = 0, \\ y(-d + x_2 - y) = 0. \end{cases}$$

If $\beta_1 > d\beta_2 + E_1$, then system (2) has three nonnegative equilibria: $P_0(0, 0, 0)$, $P_1(x_1^0, x_2^0, 0)$, $P_2(x_1^*, x_2^*, y^*)$, where

$$x_1^0 = x_2^0 = \frac{\beta_1 - E_1}{\beta_2},$$
$$x_1^* = x_2^* = \frac{\beta_1 + ad - E_1}{a + \beta_2},$$
$$y^* = \frac{\beta_1 - d\beta_2 - E_1}{a + \beta_2}.$$

Obviously, R_+^3 is invariant for system (2).

In order to discuss the permanence and extinction of system (2), we analysis the local geometric properties of the nonnegative equilibria of system (2).

The characteristic equation of equilibrium $P_0(0,0,0)$ is

$$(\lambda+1)(\lambda+E_1)(\lambda+d)-\beta_1(\lambda+d)=0.$$

Hence, $P_0(0,0,0)$ is a saddle with $\dim W^u(P_0)=1$, $\dim W^s(P_0)=2$ for $E_1<\beta_1$; $P_0(0,0,0)$ is locally asymptotically stable for $E_1>\beta_1$.

The characteristic equation of equilibrium $P_1(x_1^0, x_2^0, 0)$ is

$$(\lambda+1)(\lambda+2\beta_2 x_2^0+E_1)(\lambda+d-x_2^0) - \beta_1(\lambda+d-x_2^0)=0.$$

Hence, $P_1(x_1^0, x_2^0, 0)$ is a saddle with $\dim W^u(P_1)=1$, $\dim W^s(P_1)=2$ for $\beta_1>d\beta_2+E_1$; $P_1(x_1^0, x_2^0, 0)$ is locally aymptotically stable for $\beta_1<d\beta_2+E_1$.

The characteristic equation of equilibrium $P_2(x_1^*, x_2^*, y^*)$ is

$$(\lambda+1)(\lambda+2\beta_2 x_2^*+ay^*+E_1)(\lambda+y^*) + (\lambda+1)ax_2^*y^* - \beta_1(\lambda+y^*)=0,$$

that is

$$\lambda^3+A\lambda^2+B\lambda+C=0,$$

Where

$$A=1+\beta_1+\beta_2 x_2^*+y^*>0,$$
$$B=\beta_2 x_2^*+y^*(1+\beta_1+\beta_2 x_2^*+ax_2^*)>0,$$
$$C=x_2^*y^*(\beta_2+a)>0.$$

Obviously, $AB-C>0$. According to Routh-Hurwitz Theorem, $P_2(x_1^*, x_2^*, y^*)$ is locally asymptotically stable for $\beta_1>d\beta_2+E_1$.

In the following, we shall discuss the global properties of nonnegative equilibria.

Theorem 1 (i) If $\beta_1>d\beta_2+E_1$, then the positive equilibrium $P_2(x_1^*, x_2^*, y^*)$ of system (2) is globally asymptotically table.

(ii) If $E_1<\beta_1\le d\beta_2+E_1$, then the nonnegative equilibrium $P_1(x_1^0, x_2^0, 0)$ of system (2) is globally asymptotically stable. Hence $E_1^*=\beta_1-d\beta_2$ is the threshold of harvesting effort of system (2).

(iii) If $\beta_1\le E_1$, then the nonnegative equilibrium $P_0(0,0,0)$ of system (2) is globally aymptotically stable.

Proof (i) We define a Lyapunov function

$$V_1(x_1,x_2,y)=\sum_{i=1}^{2}\lambda_i\left(x_i-x_i^*-x_i^*\ln\frac{x_i}{x_i^*}\right) + \lambda\left(y-y^*-y^*\ln\frac{y}{y^*}\right),$$

where λ_1, λ_2, λ are suitable constants to be determined in the subsequent steps. Obviously, V_1 is a positive definite function in the region R_+^3 except at $P_2(x_1^*, x_2^*, y^*)$ where it vanishes. Further,

$$\lim_{\substack{x_i\to 0\\ y\to 0}} V_1(x_1,x_2,y)=\lim_{\substack{x_i\to\infty\\ y\to\infty}} V_1(x_1,x_2,y)=\infty.$$

The time derivative of V_1 along the solution of (2) is

$$\begin{aligned} dV_1/dt &= \lambda_1(x_1-x_1^*)\left(\frac{x_2}{x_1}-1\right) + \lambda_2(x_2-x_2^*)\left(\beta_1\frac{x_1}{x_2}-\beta_2x_2-ay-E_1\right) + \lambda(y-y^*)(-d+x_2-y) \\ &= \lambda_1(x_1-x_1^*)(x_1^*x_2-x_1x_2^*)/x_1x_1^* + \beta_1\lambda_2(x_2-x_2^*)(x_1x_2^*-x_1^*x_2)/x_2x_2^* \\ &\quad -\lambda_2\beta_2(x_2-x_2^*)^2-\lambda_2a(x_2-x_2^*)(y-y^*) + \lambda(x_2-x_2^*)(y-y^*)-\lambda(y-y^*)^2, \end{aligned}$$

Let $\lambda=\lambda_2 a$, $\lambda_1 x_2^*=\lambda_2\beta_1 x_1^*$,

$$dV_1/dt=-\frac{\lambda_1}{x_1^*x_1x_2}(x_1^*x_2-x_1x_2^*)^2 - \lambda_2\beta_2(x_2-x_2^*)^2-\lambda(y-y^*)^2\le 0.$$

Set $D_1=\{x\in \mathrm{Int}R_+^3: dV_1/dt=0\}=\{x\in\mathrm{Int}R_+^3: x_i=x_i^*, y=y^*\}=P_2$. According to LaSalle Theorem, P_2 is globally asymptotically stable for $\beta_1>d\beta_2+E_1$.

(ii) We construct the following Liapunov function

$$V_2(x_1,x_2,y)=\sum_{i=1}^{2}\lambda_i\left(x_i-x_i^0-x_i^0\ln\frac{x_i}{x_i^0}\right)+\lambda y.$$

Caculating the derivative of $V_2(t)$ along the solution of system (2), we have

$$\begin{aligned} dV_2/dt &= \lambda_1(x_1-x_1^0)\left(\frac{x_2}{x_1}-1\right) + \lambda_2(x_2-x_2^0)\left(\beta_1\frac{x_1}{x_2} - \beta_2x_2-ay-E_1\right)+\lambda y(-d+x_2-y) \\ &= \lambda_1(x_1-x_1^0)(x_1^0x_2-x_1x_2^0)/x_1x_1^0 + \beta_1\lambda_2(x_2-x_2^0)(x_1x_2^0-x_1^0x_2)/x_2x_2^0 \\ &\quad -\lambda_2\beta_2(x_2-x_2^0)^2-\lambda_2a(x_2-x_2^0)y+\lambda x_2y-\lambda dy-\lambda y^2, \end{aligned}$$

Let $\lambda_1x_2^0=\lambda_2\beta_1x_1^0$, $a\lambda_2=\lambda$,

$$dV_2(t)/dt=-\frac{\lambda_1}{x_1^0x_1x_2}(x_1^0x_2-x_1x_2^0)^2 - \lambda_2\beta_2(x_2-x_2^0)^2-\lambda(d-x_2^0)y-\lambda y^2\le 0.$$

Set $D_2=\{x\in R_+^3: dV_2/dt=0\}=P_1$. According to LaSalle Theorem, P_1 is globally asymptotically stable for $E_1<\beta_1\le d\beta_2+E_1$. We obtain $E_1^*=\beta_1-d\beta_2$ is the threshold (critical situations) of harvesting effort of system (2).

(iii) We construct the following Liapunov function $V_3(x_1,x_2,y)=\sum_{i=1}^{2}\lambda x_i+\lambda y$.

Caculating the derivative of $V_3(t)$ along the solution of system (2), we have

$$dV_3/dt=\lambda_1x_2-\lambda_1x_1+\lambda_2\beta_1x_1-\lambda_2\beta_2x_2^2-a\lambda_2x_2y-\lambda_2E_1x_2-\lambda dy+\lambda yx_2-\lambda y^2.$$

Let $\lambda_1=\lambda_2\beta_1$, $a\lambda_2=\lambda$,

$$dV_3(t)/dt=\lambda_2(\beta_1-E_1)x_2-\lambda_2\beta_2x_2^2-\lambda dy-\lambda y^2\le 0.$$

Hence, $D_3=\{x\in R_+^3: dV_3/dt=0\}=\{x\in R_+^3: x_2=y=0, x_1\ge 0\}$. If D_3 is an invariant set of system (2), by the second equation of system (2), we have $x_1=0$, $D_3=P_0(0,0,0)$. Hence, $P_0(0,0,0)$ is globally asymptotically stable for $\beta_1\le E_1$. □

According to Theorem 1, we have

Theorem 2 (i) Two species of system (1) are permanence if and only if the catchability effort satisfies

$$qE<\frac{b_1b}{r_1+b_1}-\frac{r_2r}{a_2}.$$

(ii) The predator species of system (1) is extinction and the prey species is not extinction if and only if the catchability effort satisfies

$$\frac{b_1b}{r_1+b_1}-\frac{rr_2}{a_2}\le qE<\frac{b_1b}{r_1+b_1}.$$

(iii) The two species of system (1) are extinction if and only if the catchability effort satisfies

$$qE\ge\frac{b_1b}{r_1+b_1}.$$

2 The optimal harvesting policy of system (2)

In this section, we consider the maximum sustainable yield of system (2). From the point of view of ecological managers, it may be desirable to have a unique positive equilibrium which is globally asymptotically stable, in order to plan harvesting strategies and keep sustainable development of ecosystem. Generally, the exploitation of population should be the mature population, which is more appropriate to the economic and biological views of renewable resources management.

Theorem 3 (i) If $\beta_1>2d\beta_2+ad$, the maximum sustainable yield in system (2) is

$$h_{MSY}=h(\bar{E}_1)=\frac{(\beta_1+ad)^2}{4(a+\beta_2)},$$

where $\bar{E}_1=\frac{1}{2}(\beta_1+ad)$, which is the optimal harvesting effort of system (2).

(ii) If $d\beta_2<\beta_1<2d\beta_2+ad$, the maximum sustainable yield in system (2) is

$$h_{MSY}=h(E_1^*)=d(\beta_1-d\beta_2),$$

where $E_1^*=\beta_1-d\beta_2$, which is the threshold (critical situations) of harvesting effort of system (2).

Proof Let $x_2=x_2^*$, the harvesting of system (2) is

$$h(E_1)=E_1x_2^*=\frac{E_1(\beta_1+ad-E_1)}{a+\beta_2}.$$

Caculating the derivative of $h(E_1)$ for E_1, we have

$$dh/dE_1=\frac{1}{a+\beta_2}(\beta_1+ad-2E_1),$$

hence, the solution of $dh/dE_1=0$ is $\bar{E}_1=\frac{1}{2}(\beta_1+ad)$. Comparing the two number E_1^* and $\bar{E}_1$, we have the following results:

(i) If $0<\bar{E}_1<E_1^*$, that is, $\beta_1>2d\beta_2+ad$, the corresponding maximum sustainable yield for $h(E_1)$ is

$$h_{MSY}=h(\bar{E}_1)=\frac{(\beta_1+ad)^2}{4(a+\beta_2)}.$$

(ii) If $\bar{E}_1>E_1^*>0$, that is, $d\beta_2<\beta_1<2d\beta_2+ad$, the corresponding maximum sustainable yield for $h(E_1)$ is

$$h_{MSY}=h(E_1^*)=d(\beta_1-d\beta_2).$$

Summarizing above discussion. If $d\beta_2<\beta_1<2d\beta_2+ad$, the maximum sustainable yield for $h(E_1)$ is $h_{MSY}=h(E_1^*)=d(\beta_1-d\beta_2)$. If the harvesting $h_{MSY}=h(E_1^*)$, then the nonnegative equilibria P_1 and P_2 of system (2) coincide, the nonnegative equilibrium P_1 is globally asymptotically stable by Theorem 1. Hence, the second species (predator) will be extinction eventually. Hence

$h_{MSY} = h(E_1^*) = d(\beta_1 - d\beta_2)$ is the threshold (critical situations) of harvesting for the mature prey population.

If $\beta_1 > 2d\beta_2 + ad$, then the maximum sustainable yield for $h(E_1)$ is $h_{MSY} = h(\overline{E}_1) = (\beta_1 + ad)^2/4(a + \beta_2)$. If the harvesting $h_{MSY} = h(\overline{E}_1)$, then the unique positive equilibrium P_2 of system (2) is globally asymptotically stable. Hence $h_{MSY} = h(\overline{E}_1)$ is the optimal harvesting of system (2).

Remark If $\beta_1 > 2d\beta_2 + ad$, that is, $bb_1b_2a_2 > 2rr_2b_2(r_1 + b_1) + ra_1a_2(r_1 + b_1)$, then $h(\overline{E}_1) > h(E_1^*)$. Therefore, Theorem 3 depicts an obvious fact: The maximum sustainable yield of system (1) depend on the reproduction rate of mature population, the death rates of two species and the rate predators attack at mature prey. These coefficients should satisfies: b is large, r, r_1 amd r_2 are small.

References:

[1] CHEN L. *Mathematical models and methods in ecology* [M]. Beijing: Science Press, 1988.

[2] FREEDMAN H I. *Deterministic mathematical models in population ecology* [M]. New York: Marcel Dekker, 1980.

[3] HOFBAUER J, Sigmund K. *Evolutionary games and population dynamics* [M]. Cambridge University Press, 1998.

[4] KUANG Y. *Delay differential equations with applications in population dynamics* [M]. New York: Academic Press, 1993.

[5] May R M. *Theretical ecology, principles and applications (second edition)* [M]. Oxford Blackwell Publications, 1981.

[6] MURRAY J D. *Mathematical biology (second edition)* [M]. Springer-verlag, Heidelberg, 1993.

[7] SMITH Hal L. *Monotone dynamical system: an introduction to the theory of competitive and cooperative system* [M]. American Mathematical Society, Providence, Rhode Island, 1995.

[8] CUI J, CHEN L, WANG W. *The effect of dispersal on population growth with stage-structure* [J]. Computers Math Applic, 2000, 39: 91-102.

[9] MAGNUSSON Kjartan G. *Destabilizing effect of cannibalism on a structured predator-prey system* [J]. Math. Biosci., 1999, 155: 61-75.

[10] AIELLO W G, FREEDMAN H I. *A time-delay model of single-species growth with stage structure* [J]. Math. Biosci., 1990, 101: 139-153.

[11] AIELLO W G, FREEDMAN H I, WU J. *Analysis of a model representing stage-structured population growth with state-dependent time delay* [J]. SIAM J Appl Math, 1992, 52(3): 855-869.

[12] CAO Y, Fan J, GARD T C. *The effects of state-structured population growth model* [J]. Nonlin Anal Th Meth Appl, 1992, 16(2): 95-105.

[13] CUSHING J M. *A simple model of cannibalism* [J]. Math Biosci, 1991, 107: 47.

[14] DIEKMANN O, NISBET R M, Gurney W S C, etc. *Simple mathematical models for cannibalism: a critiqueand a new approach* [J]. Math Biosci, 1986, 78: 21-46.

[15] HASTINGS A. *Cycles in cannibalistic egg-larval interactions* [J]. J Math Biol, 1987, 24: 651-666.

[16] HASTINGS A, CONSTANTINO R F. *Cannibalistic egg-larval interactions in Tribolium: an explanation for the oscillation in larval numbers* [J]. Amer Nat, 1987, 130: 36-52.

[17] CLARK C W. *Mathematical bioeconomics: the optimal management of renewable resources* [M]. 2nd ed., Wiley, New York, 1990.

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$$\Delta = \frac{1}{N(N-1)}\{\sum_{i=1}^{N}\sum_{j=1}^{N} t_i^h t_j^{h+1}(t_j^{1-h} - t_i^{1-h}) - \sum_{i=1}^{N}\sum_{j=1}^{N} t_i^{h+\frac{1}{2}} t_j^{h+\frac{1}{2}}(t_j^{1-h} - t_i^{1-h})\}$$

$$= \frac{1}{N(N-1)}\sum_{i=1}^{N}\sum_{j=1}^{N} t_i^h t_j^{h+\frac{1}{2}}(t_j^{\frac{1}{2}} - t_i^{\frac{1}{2}})(t_j^{1-h} - t_i^{1-h}),$$

因此,

当 $h > 1$ 时, $\Delta > 0$, 有 $S_x^2 > S_{f\tilde{x}}\tilde{x}_N / f_N$, 则 PPS 设计优于乘积估计设计;

当 $h < 1$ 时, $\Delta < 0$, 有 $S_x^2 < S_{f\tilde{x}}\tilde{x}_N / f_N$, 上式大于零, 此时乘积估计优于 PPS 估计;

当 $h = 1$ 时, $\Delta = 0$, 有 $S_x^2 = S_{f\tilde{x}}\tilde{x}_N / f_N$, 此时 PPS 估计和乘积估计的效率相当.

注 1　模型(1)是抽样调查理论研究中一个经常采用且十分重要模型. 如文献[1～4]中都使用过这一模型.

注 2　上面的定理说明: PPS 抽样方案的 Hansen-Hurvitz 估计与简单随机抽样方案的乘积估计其优劣于在 $1/x_i$ 本身的差异性和 $1/x_i$ 同 f_i 间的差异性的大小.

参考文献:

[1]　AGRAWAL M C. *A new predictive product estimator*[J]. Biometrika, 1989, 76(4): 822-823.

[2]　CASSES C M, SARNDAL C E, WRETMAN J H. *Some result on generalized difference estimation and generalized regression estimation for finite populations* [J]. Biometrika, 1976, 63(3): 615-620.

[3]　CASSES C M, SARNDAL C E, WRETMAN J H. *Foundations of inference in survey sampling*[M]. New York: John Wiley, 1977.

[4]　周国华, 成　平, 冯士雍. 有限总体指标值的线性函数的可容许估计[J]. 中国科学(A 辑), 1997, 27(2): 97-104.

[5]　COCHRAN W G. *Sampling technique*[M]. 3rd ed. New York: John Wiley, 1977.

[6]　冯士雍, 施锡铨. 抽样调查——理论, 方法和实践[M]. 上海: 上海科学技术出版社, 1994. 100-120.

Comparison of PPS estimation and product estimation

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Abstract: The estimation for the population mean under PPS sampling scheme and the product estimation for population mean under simple random sampling scheme are two often methods in practice. Their theoretical research is very rich too. In the present paper, the efficiency of this two estimation is comparied in case that interest index and size index being negative related.

Key words: PPS estimation; product estimation; super population model

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阶段结构捕食系统的最优收获策略

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摘　要: 在生物经济即可更新资源管理中, 捕食系统起着非常重要的作用. 本文建立一个具阶段结构和捕食关系的两种群生物经济模型, 分别获得了一个或两个种群绝灭和两种群持久生存的充分必要条件, 也获得了保持生态环境持续发展的最优收获策略和收获成年种群的阈值, 并给出了所得结果的实际意义.

关键词: 生物经济模型; 持续生存; 最优收获策略

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