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·基础理论研究·

Persistence and global stability for nonautonomous predator-prey system with diffusion and time delay

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Abstract: A nonautonomous predator-prey model with diffusion and continuous time delay is studied, where all parameters are time-dependent. The system, which is composed of two Lotka - Volterra patches, has two species: one can diffuse between two patches, but the other is confined to one patch and can not diffuse. The system is uniformly persistent under appropriate conditions. Furthermore, sufficient conditions are estabished for global stability of the system.

Key words: uniform persistence; diffusion; time delay; global asymptotic stability

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1 Introduction

One of the most interesting questions in mathematical biology concerns the survial of species in ecological models. In this paper, we consider a nonautomous system composed of two species predator-prey with diffusion and continuous time delay. LEVIN[1], KISHIMOTO[2] and TAKEUCHI[3] studied these kinds of models. SONG and CHEN[4], LIU and WU[5] extended the autonomous Lotka-Volterra system to a two-species nonautomous diffusion Lotka-Volterra system with time delay, In this paper, we assume that all the coefficients in the system depend on them.

2 Model and background concept

In this paper, we consider the following Lotka-Volterra population model:

$$\begin{cases} \dot{x}_1 = x_1(a_1(t) - b_1(t)x_1 - c(t)y) \\ \qquad + D_1(t)(x_2 - x_1), \\ \dot{x}_2 = x_2(a_2(t) - b_2(t)x_2) \\ \qquad + D_2(t)(x_1 - x_2), \\ \dot{y} = y(-d(t) + e(t)x_1 - q(t)y \\ \qquad - \beta(t)\int_{-\tau}^{0} k(s)y(t+s)\,ds) \end{cases} \quad (1)$$

Where x_1 and y are the population density of prey species x and predator species y in patch 1, and x_2 is the density of prey species x in patch 2. Predator species y is confined to patch 1, while the prey species x can diffuse between two patches. $D_{i(t)}$ $(i = 1, 2)$ are diffusion coefficents of species x.

2 Persistence

In system (1), we always assume that the following.

Assumption (H_1) $a_i(t)$, $b_i(t)$, $d_i(t)$, $(i = 1, 2)$, $c(t)$, $d(t)$, $e(t)$, $q(t)$, and $\beta(t)$ are continuous and strictly positive functions, which statisfy

$$\min_{i=1,2}\{a_i^l, b_i^l, d_i^l, c^l, e^l, d^l, q^l, \beta^l\} > 0,$$

$$\max_{i=1,2}\{a_i^m, b_i^m, d_i^m, c^m, e^m, d^m, q^m, \beta^m\} < \infty.$$

Where we let $f^l = \inf\{f(t) : t \in R\}$, $f^m = \sup\{f(t) : t \in R\}$. for a continuous and bounded function $f(t)$.

Assumption (H_2) $k(s) \geq 0$, on $[-\tau, 0]$, $(0 \leq \tau < \infty)$ and $k(s)$ is a piecewise continuous and normalized function such that $\int_{-\tau}^{0} k(s)\,ds = 1$.

We adopt the following notations and concept throughout this paper.

Let $x = (x_1, x_2, y) \in R_+^3 = \{x \in R^3 : x_i \geq 0 (i = 1, 2), y \geq 0\}$. Denote $x > 0$, if $x \in \ln R_+^3$, for ecological reasons, we consider system (1) only

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in $\ln R_+^3$.

Let $C^+ = C([-\tau, 0]; R_3^+)$ denote the Banach space of all nonnegative continuous functions with

$$\|\Phi\| = \sup_{s \in [-\tau, 0]} |\Phi(s)|, \text{for } \Phi \in C^+.$$

Then, if we choose the initial function space of system (1) to be C^+, it is easy to see that, for any $\Phi = (\phi_1, \phi_2, \phi_3) \in C^+$ and $\Phi(0) > 0$, there exists $\alpha \in (0, \infty)$ and unique solution $x(t, \Phi)$ of system (1) on $[-\tau, \alpha]$, which remains positive for all $t \in [0, \alpha]$, such solutions of (1) are called positive solutions. Hence, in the reast of this paper, we always assume that

$$\Phi \in C^+, \Phi(0) > 0 \tag{2}$$

Definition System (1) is said to be uniformly persistent if there exists a compact region $D \subset \int R_+^3)$ such that every solution $x(t) = (x_1, x_2, y)$ of (1) with initial condition (2) eventually enters and remains in the region D.

In this paper, a positive solution of (1) is called globally asymptotically stable if it is stable and attracts all positive solutions.

We let

$$M_1^* = \max\{\frac{a_1^m}{b_1^l}, \frac{a_2^m}{b_2^l}\}, M_2^* = \frac{e^m M}{q^e},$$

$$m_1^* = \min\{(a_1^l - c^m M_2^*)/b_1^m, a_2^l/b_2^m\}.$$

We have

Lemma 1 Suppose that system (1) satisfies the following,

Assumption (H_3), $(a_1^l q^l)/(c^m e^m) > m_1^*$,

Assumption (H_4), $(e^l m_1^*) > d^m + \beta^m m_2^*$.

Then sysem (1) is uniformly persistent.

We do not prove this lemma here, see [4].

3 Global asymptotic stability

In this section, we derive sufficient conditions which guarantee that any positive solution of system (1) is globally asymptotically stable.

Theorem 1 In addition to $(H_1) \sim (H_4)$, assume further that system (1) satisfies the following.

Assumption (H_5) $b_1(t) \geq e(t) + \frac{D_2(t)}{m_1^*}$, and

$$\lim_{t\to\infty} \int_0^t (b_1(t) - e(t) - \frac{D_2(t)}{m_1^*}) dt = \infty;$$

$$b_2(t) \geq \frac{D_1(t)}{m_1^*},$$

and $\lim_{t\to\infty} \int_0^t (b_2(t) - \frac{D_2(t)}{m_1^*}) dt = \infty$;

$$q(t) \geq c_1(t) + \beta^m,$$

and $\lim_{t\to\infty} \int_0^t (q(t) - c(t) - \beta^m) dt = \infty$.

Then any positive solution of (1) is globally asymptotically stable.

Proof For two arbitrary nontrivial positive solutions $x(t) = (x_1, x_2, y)$ and $u(t) = (u_1, u_2, v)$ of (1). we have from uniform persistence of (1) that there exists positive constants m_i and M_i $(i = 1, 2)$ such that for all $t \geq t^*$ (t^* sufficient large),

$$\begin{cases} 0 < m_1 \leq x_i(t) \leq M_1 \ (i = 1, 2), \\ 0 < m_2 \leq y(t) \leq M_2, \\ 0 < m_1 \leq u_i(t) \leq M_1 \ (i = 1, 2), \\ 0 < m_2 \leq V(t) \leq M_2, \end{cases} \tag{3}$$

We define

$$\tilde{x}_i = \ln x_i, \tilde{y} = \ln y,$$

$$\tilde{u}_i = \ln u_i, \tilde{v} = \ln v \ (i = ,1, 2).$$

Consider the following Lyapunov functional

$$\begin{aligned} L(t) = &\sum_{i=1}^{2} |\tilde{x}_i(t) - \tilde{u}(t)| \\ &+ |\tilde{y}(t) - \tilde{v}(t)| \\ &+ \beta^m \int_{-\tau}^{0} k(s) \int_{t+s}^{t} |y_1(\theta) - v(\theta)| d\theta ds. \end{aligned}$$

Now we calculate and estimate the upper-right derivate of $L(t)$ along the solutions of (1)

$$\begin{aligned} D^+ L(t) \leq &- (b_1(t) - e(t)) |x_1 - u_1| \\ &- b_2(t) |x_2 - u_2| \\ &- (q(t) - c(t)) |y(t) - v(t)| \\ &+ \beta^m \int_{-\tau}^{0} k(s) |y(t+s) - v(t+s)| ds \\ &- \beta^m \int_{-\tau}^{0} k(s) |y(t) - v(t)| ds \\ &+ \tilde{D}_1(t) + \tilde{D}_2(t), \end{aligned}$$

where

$$\tilde{D}_1(t) = D_1(t)(\frac{x_2}{x_1} - \frac{u_2}{u_1}) \operatorname{sgn}(x_1 - u_1),$$

$$\tilde{D}_2(t) = D_2(t)(\frac{x_1}{x_2} - \frac{u_1}{u_2}) \operatorname{sgn}(x_2 - u_2).$$

Now consider $\tilde{D}_1(t)$ for the following two cases.

(a) $x_1 \geq u_1$ and $t \geq t^*$, then

$$\widetilde{D}_1(t) \le \frac{D_1(t)}{u_1(t)}(x_2 - u_2) \le \frac{D_1(t)}{m_1}|x_2 - u_2|.$$

(b) $x_1 < u_1$ and $t \ge t^*$, then

$$\widetilde{D}_1(t) \le \frac{D_1(t)}{x_1(t)}(u_2 - x_2) \le \frac{D_1(t)}{m_1}|x_2 - u_2|.$$

From (a), (b), we have

$$\widetilde{D}_1(t) \le \frac{D_1(t)}{m_1}(u_2 - x_2) \quad ,\text{for } t \ge t^*,$$

Consider for $\widetilde{D}_2(t)$ in the same way, we can obtain

$$\widetilde{D}_2(t) \le \frac{D_2(t)}{m_1}(x_1 - u_1) \text{ ,for } t \ge t^*,$$

Hence, we have

$$D^+ L(t) \le -(b_1(t) - e(t) - \frac{D_2(t)}{m_1})|x_1 - u_1| - (b_2(t) - \frac{D_1(t)}{m_1})|x_2 - u_2| - (q(t) - c(t) - \beta^n)|y - v|.$$

From the proof of Lemma 1, we know that m_1 can be close m_1^* sufficiently, so according to Assumption (H_5), we have

$$b_1(t) \ge e(t) + \frac{D_1(t)}{m_1},$$

and

$$\lim_{t\to\infty}\int_0^t (b_1(t) - e(t) - \frac{D_1(t)}{m_1})\,dt = \infty;$$

$$b_2(t) \ge \frac{D_2(t)}{m_1},$$

and

$$\lim_{t\to\infty}\int_0^t (b_2(t) - \frac{D_2(t)}{m_1})\,dt = \infty \tag{5}$$

We let

$$\widetilde{L}(t) = \sum_{i=1}^{2}|x_i - u_i| + |y - v|.$$

From (4), we can obtain

$$\widetilde{L}(t) \le 4M_1 + 2M_2, \text{ for } t \ge t^*.$$

If $\lim_{t\to\infty}\widetilde{L}(t) = 3d$, $(d > 0)$, we have

$$\lim_{t\to\infty}|x_i(t) - u_i(t)| \ge d, (i = 1, 2)$$

or

$$\lim_{t\to\infty}|y(t) - v(t)| \ge d.$$

So, without loss of any generality, we can assume that $\lim_{t\to\infty}|x_1 - u_1| \ge d$, then we obtain

$$D^+ L_1(t) \le -(b_1(t) - e(t) - \frac{D_2(t)}{m_1})d \tag{6}$$

An integration (6) leads to

$$L(t) - L(t^*) \le -d\int_0^t (b_1(t) - e(t) - \frac{D_2(t)}{m_1})\,dt,$$

therefore

$$L(t) + d\int_0^t (b_1(t) - e(t) - \frac{D_2(t)}{m_1})\,dt \le L(t^*) \tag{7}$$

From (5), we have

$$\lim_{t\to\infty}\int_0^t (b_1(t) - e(t) - \frac{D_2(t)}{m_1})\,dt = \infty$$

We obtain that (7) does not hold, so, $d = 0$, there exists a sequence of number $\{t_n\}$, which satisfies $0 < t_1 < t_2 < \cdots < t_n < \cdots$, such that

$$\lim_{t\to\infty}\widetilde{L}(t_n) = 0$$

and

$$\begin{cases} m_1 \le x_i(t_n) \le M_1\ (i = 1, 2), \\ m_2 \le y(t_n) \le M_2; \\ m_1 \le u_i(t_n) \le M_1\ (i = 1, 2), \\ m_2 \le v(t_n) \le M_2. \end{cases} \tag{8}$$

From (8), we can obtain $\lim_{t\to\infty}\widetilde{L}(t_n) = 0$.

From (4), we know that $L(t)$ monotonic decrease.

So, we get $\lim_{t\to\infty}L(t) = 0$, we can say

$$\lim_{t\to\infty}|x_i - u_i| = 0, (i = 1, 2), \lim_{n\to\infty}|\overline{y} - \overline{v}| = 0. \tag{9}$$

From (3), (9), we have

$$\lim_{t\to\infty}|x_i - u_i| = 0, (i = 1, 2), \lim_{t\to\infty}|y - v| = 0.$$

This result implies that any positive solution of (1) is stable and attracts all positive solution of (1). The proof is complete.

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带有扩散和时滞的非自治捕食系统的持续生存和全局稳定性

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摘　要:讨论带有扩散和时滞的非自治捕食系统,有两种种群:一个能在两个斑块中自由扩散;而另一个被限定一个斑块中不能扩散,在一定的条件下,得到系统持续生存和全局稳定性.

关键词:持续生存;扩散;时滞;全局渐近稳定性

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The rings characterized by singular modules

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Abstract: Some characterizations of regular rings and semisimple rings via singular modules are given, and some characterizations of field over commutative domain rings are obtained.

Key words: singular module; semisimple rings; regular ring; flat module; coflat module

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