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· 基础理论研究 ·

Oscillation criteria for second order neutral functional differentia equations

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Abstract: The second order neutral functional differential equations of mixed type:

$$\frac{d^2}{dt^2}[x(t)+cx(t-h)+c^*x(t+h^*)]+qx(t-g)+px(t+g^*)=0$$

is studied, where c, c^*, h, h^*, p, q are real numbers, g and g^* are positive constants, and some new criterias for the oscillation of those equations are established. The results in this paper improve all theorems in [1].

Key words: differential equation; neutral; oscilliation

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1 Introduction

Consider the second order neutral functional equation

$$\frac{d^2}{dt^2}[x(t)+cx(t-h)+c^*x(t+h^*)] + qx(t-g)+px(t+g^*)=0,$$

where c, c^*, h, h^*, p, q are real numbers, g and g^* are positive constants.

The purpose of this article is to establish some easily verifiable sufficient conditions, involving the coefficients and the arguments only. More precisely, we study the oscillatory character of the neutral equations:

$$(x(t)+cx(t-h)-c^*x(t+h^*))^{(2)} = qx(t-g)+px(t+g^*) \quad (1)$$

$$(x(t)+cx(t+h)-c^*x(t+h^*))^{(2)} = qx(t-g)+px(t+g^*) \quad (2)$$

$$(x(t)+cx(t-h)+c^*x(t+h^*))^{(2)} = qx(t-g)+px(t+g^*) \quad (3)$$

and

$$(x(t)-cx(t-h)-c^*x(t+h^*))^{(2)} + qx(t-g)+px(t+g^*)=0 \quad (4)$$

where c, c^*, h and h^* are nonnegative constants, g, g^*, p, and q are positive real numbers.

The problem of oscillatory behavior of solutions of neutral differential equations is of both theoretical and practical interest. See for example [4～5], and the references cited therein.

As is customary, a solution is called oscillatory if it has arbitrarily large zeros and nonoscillatory if it is eventually positive or eventually negative. Equation is called oscillatory if all its solutions are oscillatory.

In the sequel all functional inequalities that we write are assumed to hold eventually, that is, for all sufficiently large t.

Our results in this paper improve all theorems in [1].

2 Main results

In the following theorem we consider the neutral Eq. (1) and we obtain a sufficient condition under which all solutions of Eq. (1) oscillatory.

Theorem 1 If $g>h$,

$$p>\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*+h)^2}e^{-2},$$

and

$$q>\frac{2^2}{g^2}e^{-2}+c\frac{2^2}{(g-h)^2}e^{-2},$$

then Eq. (1) is oscillatory.

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Biographies: CHENG Jin-fa (1967-), male, Leping, Jiangxi, Asso. Prof., D., specialization of k-quasiconformal mapping, functional differential equations

Proof Eq. (1) is equivalent to

$$(x(t-g^*)+cx(t-h-g^*) - c^*x(t+h^*-g^*))^{(2)} = qx(t-g-g^*)+px(t) \qquad (1)'$$

or

$$(x(t+g)+cx(t-h+g) - c^*x(t+h^*+g))^{(2)} = qx(t)+px(t+g^*+g) \qquad (1)''$$

for Eq. (1)′ or Eq. (1)″, its characteristic equation polynomial is

$$F(\lambda)=\lambda^2e^{-\lambda g^*}+c\lambda^2e^{-\lambda(h+g^*)} - c^*\lambda^2e^{\lambda(h^*-g^*)}-p-qe^{-\lambda(g+g^*)},$$

or

$$F(\lambda)=\lambda^2e^{\lambda g}+c\lambda^2e^{-\lambda(h-g)} - c^*\lambda^2e^{\lambda(h^*+g)}-q-pq^{\lambda(g^*+g)}$$

for $F(\lambda)$, when $\lambda>0$, then

$$F(\lambda)<\lambda^2e^{-\lambda g^*}+c\lambda^2e^{-\lambda(h+g^*)}-p,$$

when $\lambda=\frac{2}{g^*}$, $\lambda^2e^{-\lambda g^*}$ attains its maximum. when $\lambda=\frac{2}{h+g^*}$, $\lambda^2e^{-\lambda(h+g^*)}$ attains its maximum.

Therefore,

$$F(\lambda)<\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*+h)^2}e^{-2}-p.$$

Provided

$$p>\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*+h)^2}e^{-2},$$

then $F(\lambda)<0$.

For $F(\lambda)$, if $\lambda<0$, then

$$F(\lambda)<\lambda^2e^{\lambda g}+c\lambda^2e^{-\lambda(h-g)}-q,$$

when $\lambda=-\frac{2}{g}$, $\lambda^2e^{\lambda g}$ attains its maximum. when $\lambda=-\frac{2}{g-h}$, $\lambda^2e^{\lambda(g-h)}$ attains its maximum.

Provided

$$q>\frac{2^2}{g^2}e^{-2}+c\frac{2^2}{(g-h)^2}e^{-2},$$

we have $F(\lambda)<0$.

In a word, characteristic equation $F(\lambda)=0$ (or $F(\lambda)=0$) has no real roots. So Eq. (1) is oscillatory. This completes the proof. □

The following criterion is concerned with the oscillatory behavior of Eq. (2).

Theorem 2 If $g^*>h$,

$$p>\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*-h)^2}e^{-2}$$

and

$$q>\frac{2^2}{g^2}e^{-2}+c\frac{2^2}{(g+h)^2}e^{-2},$$

then Eq. (2) is oscillatory.

Proof Eq. (2) is equivalent to

$$(x(t-g^*)+cx(t+h-g^*) - c^*x(t+h^*-g^*))^{(2)} = qx(t-g-g^*)+px(t) \qquad (2)'$$

or

$$(x(t+g)+cx(t+h+g) - c^*x(t+h^*+g))^{(2)} = qx(t)+px(t+g^*+g) \qquad (2)''$$

for Eq. (2)′ or Eq. (2)″, its characteristic equation polynomial is

$$F(\lambda)=\lambda^2e^{-\lambda g^*}+c\lambda^2e^{-\lambda(g^*-h)} - c^*\lambda^2e^{\lambda(h^*-g^*)}-p-qe^{-\lambda(g+g^*)}$$

or

$$F(\lambda)=\lambda^2e^{\lambda g}+c\lambda^2e^{\lambda(h+g)} - c^*\lambda^2e^{\lambda(h^*+g)}-q-pe^{\lambda(g^*+g)}$$

for $F(\lambda)$, when $\lambda>0$, then

$$F(\lambda)<\lambda^2e^{-\lambda g^*}+c\lambda^2e^{-\lambda(g^*-h)}-p$$

when $\lambda=\frac{2}{g^*}$, $\lambda^2e^{-\lambda g^*}$ attains its maximum. when $\lambda=\frac{2}{g^*-h}$, $\lambda^2e^{-\lambda(g^*-h)}$ attains its maximum.

Therefore,

$$F(\lambda)<\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*-h)^2}e^{-2}-p.$$

Provided

$$p>\frac{2^2}{g^{*2}}e^{-2}+c\frac{2^2}{(g^*-h)^2}e^{-2}$$

then $F(\lambda)<0$.

For $F(\lambda)$, if $\lambda<0$, then

$$F(\lambda)<\lambda^2e^{\lambda g}+c\lambda^2e^{\lambda(h+g)}-q$$

when $\lambda=-\frac{2}{g}$, $\lambda^2e^{\lambda g}$ attains its maximum. when $\lambda=-\frac{2}{g+h}$, $\lambda^2e^{\lambda(g+h)}$ attains its maximum.

Provided

$$q>\frac{2^2}{g^2}e^{-2}+c\frac{2^2}{(g+h)^2}e^{-2}$$

we have $F(\lambda)<0$.

In a word, characteristic equation $F(\lambda)=0$ (or

$F(\lambda)=0$) has no real roots So Eq. (2) is oscillatory. This completes the proof. □

Next, we present a result which deals with the oscillatory character of Eq. (3).

Theorem 3　If $g > h, g^* > h^*$,

$$p > \frac{2^2}{g^{*2}}e^{-2} + c\frac{2^2}{(g^*+h)^2}e^{-2} + c^*\frac{2^2}{(g^*-h^*)^2}e^{-2}$$

and

$$q > \frac{2^2}{g^2}e^{-2} + c\frac{2^2}{(g-h)^2}e^{-2} + c^*\frac{2^2}{(g+h^*)^2}$$

then Eq. (3) is oscillatory.

Proof　Eq. (3) is equivalent to

$$(x(t-g^*) + cx(t-h-g^*) + c^*x(t+h^*-g^*))^{(2)} = qx(t-g-g^*) + px(t) \qquad (3)'$$

or

$$(x(t+g) + cx(t-h+g) + c^*x(t+h^*+g))^{(2)} = qx(t) + px(t+g^*+g), \qquad (3)''$$

for Eq. (3)' or Eq. (3)'', its characteristic equation polynomial is

$$F(\lambda) = \lambda^2 e^{-\lambda g^*} + c\lambda^2 e^{-\lambda(g^*+h)} + c^*\lambda^2 e^{\lambda(h^*-g^*)} - p - qe^{-\lambda(g+g^*)}$$

or

$$F(\lambda) = \lambda^2 e^{\lambda g} + c\lambda^2 e^{\lambda(-h+g)} + c^*\lambda^2 e^{\lambda(h^*+g)} - q - pe^{\lambda(g^*+g)}$$

for $F(\lambda)$, when $\lambda > 0$, then

$$F(\lambda) < \lambda^2 e^{-\lambda g^*} + c\lambda^2 e^{-\lambda(g^*+h)} + c^*\lambda^2 e^{-(g^*-h^*)} - p$$

when $\lambda = \frac{2}{g^*}$, $\lambda^2 e^{-\lambda g^*}$ attains its maximum. when $\lambda = \frac{2}{g^*+h}$, $\lambda^2 e^{-\lambda(g^*+h)}$ attains its maximum. And when $\lambda = \frac{2}{g^*-h}$, $\lambda^2 e^{-\lambda(g^*-h)}$ attains its maximum.

Therefore,

$$F(\lambda) < \frac{2^2}{g^{*2}}e^{-2} + c\frac{2^2}{(g^*-h)^2}e^{-2} + c^*\frac{2^2}{(g^*-h^*)^2} - p.$$

Provided

$$p > \frac{2^2}{g^{*2}}e^{-2} + c\frac{2^2}{(g^*-h)^2}e^{-2} + c^*\frac{2^2}{(g^*-h^*)^2}.$$

then $F(\lambda) < 0$.

For $F(\lambda)$, if $\lambda < 0$, then

$$F(\lambda) < \lambda^2 e^{\lambda g} + c\lambda^2 e^{\lambda(-h+g)} + c^*\lambda^2 e^{\lambda(g+h^*)} - q$$

when $\lambda = -\frac{2}{g}$, $\lambda^2 e^{\lambda g}$ attains its maximum. when $\lambda = -\frac{2}{g-h}$, $\lambda^2 e^{\lambda(g-h)}$ attains its maximum. And when $\lambda = -\frac{2}{g+h^*}$, $\lambda^2 e^{\lambda(g+h^*)}$ attains its maximum.

Provided

$$q > \frac{2^2}{g^2}e^{-2} + c\frac{2^2}{(g-h)^2}e^{-2} + c^*\frac{2^2}{(g+h^*)^2}e^{-2},$$

we have $F(\lambda) < 0$.

In a word, characteristic equation $F(\lambda)=0$ (or $F(\lambda)=0$) has no real roots So Eq. (3) is oscillatory. This completes the proof. □

Finally, we will present an oscillation for Eq. (4) when c and c^* are nonnegative constants.

Theorem 4　If $g > h, g^* > h^*$,

$$p > c\frac{2^2}{(h+g^*)^2}e^{-2} + c^*\frac{2^2}{(g^*-h^*)^2}e^{-2}$$

and

$$q > c\frac{2^2}{(g-h)^2}e^{-2} + c^*\frac{2^2}{(g+h^*)^2}e^{-2},$$

then Eq. (4) is oscillatory.

Proof　Eq. (4) is equivalent to

$$(x(t-g^*) - cx(t-h-g^*) - c^*x(t+h^*-g^*))^{(2)} + qx(t-g-g^*) + px(t) = 0 \qquad (4)'$$

or

$$(x(t+g) - cx(t-h+g) - c^*x(t+h^*+g))^{(2)} + qx(t) + px(t+g^*+g) = 0 \qquad (4)''$$

for Eq. (4)' or Eq. (4)'', its characteristic equation polynomial is

$$F(\lambda) = \lambda^2 e^{-\lambda g^*} - c\lambda^2 e^{-\lambda(g^*+h)} - c^*\lambda^2 e^{\lambda(h^*-g^*)} + p + qe^{-\lambda(g+g^*)}$$

or

$$F(\lambda) = \lambda^2 e^{\lambda g} - c\lambda^2 e^{\lambda(-h+g)} - c^*\lambda^2 e^{\lambda(h^*+g)} + q + pe^{\lambda(g^*+g)}$$

for $F(\lambda)$, when $\lambda > 0$, then

$$F(\lambda) > -c\lambda^2 e^{-\lambda(h+g^*)} - c^*\lambda^2 e^{-\lambda(g^*-h^*)} + p$$

when $\lambda = \frac{2}{g^*+h}$, $\lambda^2 e^{-\lambda(h+g^*)}$ attains its maximum. when $\lambda = \frac{2}{g^*-h^*}$, $\lambda^2 e^{-\lambda(g^*-h^*)}$ attains its maximum.

Therefore,

$$F(\lambda) > -c\frac{2^2}{(h+g^*)^2}e^{-2} - c^*\frac{2^2}{(g^*-h^*)^2}e^{-2} + p.$$

Provided

$$p > c\frac{2^2}{(h+g^*)^2}e^{-2} + c^*\frac{2^2}{(g^*-h^*)^2}e^{-2},$$

then $F(\lambda) > 0$.

For $F(\lambda)$, if $\lambda < 0$, then

$$F(\lambda) > -c\lambda^2 e^{\lambda(g-h)} - c^*\lambda^2 e^{\lambda(h^*+g)} + q$$

when $\lambda = -\frac{2}{g-h}$, $\lambda^2 e^{\lambda(g-h)}$ attains its maximum.

when $\lambda = -\frac{2}{g+h^*}$, $\lambda^2 e^{\lambda(g+h^*)}$ attains its maximum.

Provided

$$q > c\frac{2^2}{(g-h)^2}e^{-2} + c^*\frac{2^2}{(g+h^*)^2}e^{-2},$$

we have $F(\lambda) > 0$.

In a word, characteristic equation $F(\lambda) = 0$ (or $F(\lambda) = 0$) has no real roots. So Eq. (4) is oscillatory. This completes the proof. □

Remark: 1. Our theorem 1～4 actually improve the theorem 1～4 offered by GRACE S. R. and LALL IB. S. in[1].

2. Our technique here is presented in such a way that it can be extended in a straight forward manner to higher order differential equations and difference equations.

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二阶泛函微分方程解的振动性判别准则

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摘　要: 研究了二阶混合中立型泛函数微分方程 $\frac{d^2}{dt^2}[x(t)+cx(t-h)+c^*x(t+h^*)]+qx(t-g)+px(t+g^*)=0$, 这里 c, c^*, h, h^*, p, q 是实数, g 和 g^* 是正常数. 并对其解的振动性建立了若干判别准则. 本文结果改进了文[1]中所有定理.

关键词: 微分方程; 中立型; 振动性

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