Theorem 3.9 If q is odd, then there exist HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

The main result of this paper becomes evident now.

Theorem 3.10 Suppose that $n\equiv 0 \pmod 4$, for $u\geqslant 2$, there exist HSOLS($2^n u^1$) if and only if $n\geqslant 1+u$.

Proof The necessity comes fron Theorem 1.1. The sufficiency comes from Theorem 3.5 and Theorem 3.9.

Acknowledgement The author is thankful to Professor Zhu Lie for introducing him to this work.

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型为$2^n u^1$的带洞自正交拉丁方当$n\equiv 0 \pmod 4$时的存在性

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摘　要　本文证明了当 $n\equiv 0 \pmod 4$且 $u\geqslant 2$时，型为$2^n u^1$的带洞自正交拉丁方存在的充分必要条件为 $n\geqslant 1+u$.

关键词　带洞自正交拉丁方；对称截态；膨胀构作法

分类号　O157.2

Exitence of SOLS with Holes of Type $2^n u^1$ for $n \equiv 0 \pmod 4$

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Abstract In this paper, it is shown that for $n \equiv 0 \pmod 4$ and $u \geq 2$, an HSOLS $(2^n u^1)$ exists if and only if $n \geq 1+u$.

Keywords Holey self-orthogonal latin square; Symmetric transversal; Inflation Construction

1 INTRODUCTION

For formal definition of MOLS with holes, the reader is refered to [7]. Let HMOLS$(h_1^{n_1} h_2^{n_2} \cdots h_k^{n_k})$ denote a pair of MOLS of order $\sum_{i=1}^{K} n_i h_i$ from which these subsquares are disjoint and spanning. The type of the HMOLS is defined to be $h_1^{n_1} h_2^{n_2} \cdots h_k^{n_k}$ (it is also convenient to think of the type as a multiset). An HSOLS$(h_1^{n_1} h_1^{n_1} \cdots h_k^{n_k})$ is defined to be an HMOLS$(h_1^{n_1} h_2^{n_2} \cdots h_k^{n_k})$ in which the two squares are mutal transposes.

The following results concerning HSOLS(h^n) and HSOLS$(h^n u^1)$ have been proved.

Theorem 1. 1

(1) [3] There exists an HSOLS(1^n) if and only if $n \geq 4, n \neq 6$.

(2) [14,16] For $h \geq 2$, there exists an HSOLS(h^n) if and only if $n \geq 4$.

(3) [19] There exist HSOLS$(1^{v-u} u^1)$ if $v \geq 3u+1$ and $(v,u) \neq (6,1)$ or $(3u+2,u)$, where $u \in \{2,4,6,8,10,14,16,18,20,22,26,28,32,34,46\}$.

(4) [16,17] There exists an HSOLS$(2^n 3^1)$ if and only if $n \geq 4$.

(5) [17] For $u=3,4,5,9$, there exist HSOLS$(2^n u^1)$ if and only if $n \geq 1+u$.

(6) [17] Let $u \geq 1$, If $n \geq 4\lceil u/3 \rceil + 10$ ($\lceil x \rceil$ denotes the smallest integer $\geq x$), then there exist HSOLS $(2^n u^1)$.

Theorem 1. 2 [16] If there exist HSOLS$(h^n u^1)$, then $n \geq 1+2u/h$.

HSOLS has been very useful in recursive constructions of various combinatorial designs, such as 2-perfect m-cycle systems[12], intersections of transversal designs[6], and skew Room frames[5]. For some results on HMOLS, we refer to [1,2,7,11], [13,15], [18]. Paper [17] has shown that for any positive integer u, an HSOLS$(2^n u^1)$ exists if $n \geq 4\lceil u/3 \rceil + 10$ and also shown that for $u=3,4$ and 5, HSOLS$(2^n u^1)$ exists if and only if $n \geq 1+u$. and conjectured that for $u>2$, an HSOLS$(2^n u^1)$ exists if and only if $n \geq 1+u$. In this paper, we shown that the conjecture is true when $n \equiv 0 \pmod 4$.

2 CONSTRUCTIONS FOR HSOLS

The following lemma is a modification of the starter-adder type construction[11].

Lemma 2. 1 Let $\theta = (\Phi, a_{o1}, a_{o2}, \cdots a_{o(n-1)}, \Phi, a_{o(n+1)}, \cdots, a_{o(2n-1)})$ be a vecter of length $2n$ with entries in $(Z_{2n} \setminus \{0,n\}) \cup X$, where $X = \{x_1, x_2, \cdots, x_u\}$, "$\Phi$" means that the cell it occupies is empty. Let $f = (a_{ox_1}, a_{ox_2}, \cdots, a_{ox_u})$ and $g = (a_{ox_1}, a_{ox_2}, \cdots, a_{ox_u})$ be vectors which are used to construct an array $A = (a_{ij})$ of order $2n+u$ with n empty subarrays of order two and one empty subarray of order u having row and column indices and entries in $Z_{2n} \cup X$. The array is constructed as follows, where all elements including indices are calculated modulo $2n$ and the x_i act as "infinite" elements.

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(1) If $a_{ij}=\Phi, 0\leqslant i,j\leqslant 2n-1$, then $a_{(i+1)(j+1)}=\Phi$

(2) If $a_{ij}\in Z_{2n}, 0\leqslant i,j\leqslant 2n-1$, then $a_{(i+1)(j+1)}=a_{ij}+1$

(3) If $a_{ij}\in X, 0\leqslant i,j\leqslant 2n-1$, then $a_{(i+1)(j+1)}=a_{ij}$.

(4) If $0\leqslant i\leqslant 2n-1$, *and* $j\in X$, then $a_{(i+1)(j+1)}=a_{ij}+1$.

(5) If $0\leqslant j\leqslant 2n-1$ and $i\in X$, then $a_{i(j+1)}=a_{ij}+1$.

Conditions can be described for the vectors e, f and g so that the array as constructed is an FSOLS ($2^n u^1$). *However, we shall simply give the vectors and the reader can check for himself that they do yield the desifed FSOLS*($2^n u^1$).

Lemma2.2 If there exist HSOLS $((2n_1)^1(2n_2)^1\cdots(2n_k)^1h^1)$ and HSOLS$(2^n v^1)$ for $1\leqslant i\leqslant k$, then there exist HSOLS$(2^n u^1)$, where $n=\sum_{i=1}^{k} n_i$ and $u=h+v$.

The following recursive construction rely on the other orthogonal arrays and on information regarding the location of transversals in certain latin squares. To this end we need more notations.

An MOLS(v) is a pair of mutual orthogonal latin square of order v and an IMOLS(v,n), a pair of **incomplete** mutual orthogonal latin squares, is a pair of mutual orthogonal latin squares of order v each with a common subarray of order n missing (that is, with an empty subarray of order n positioned at the same location in each square), in porticular, an ISOLS $(v,0)$ is an SOLS(v).

Let $L=(a_{ij})$ be a latin square of order n. we call two transversals **disjonit** if they have no cell in common. A transversal is said to be **symmetlic** if $(i,j)\in T$ implies $(j,i)\in T$. A pair of transversals T and S are said to be symmetric if $(i,j)\in T$ implies $(j,i)\in S$

The following theorems provide the "ingredients" for constructing HSOLSs in the next seotion.

Theorem 2.3 [4] There exists an MOLS(v) for all values of $v, v\neq 2,6$.

Theorem 2.4 [10] There exists an IMOLS(v,n) for all values of v and n satisrying $v\geqslant 3n$ except that an IMOLS(6,1) does not exist.

Theorem 2.5 [9] If $q\geqslant 5$ is an odd prime power, then there exists an HSOLS(1^q) with $q-1$ disjoint transversals and occuring as $(q-1)/2$ pairs of symmetric transversals.

Theorem 2.6 [8] For all even q, $q\notin\{2,6,10,14,46,54,58,62,66,70\}$, there exist HSOLS$(1^q)$ with $q-1$ disjoint symmetric transversals.

The following recursive construction is referred to as the Inflation Construction.

Lemma 2.7 [19, Construction 3.3] Suppose there is an HSOLS(1^q) which has $m+2n$ disjoint transversals, m of them being symmetric and the rest being n symmetric pairs, For $1\leqslant i\leqslant m$ and $1\leqslant j\leqslant n$, let $v_i\geqslant 0$ and $w_j\geqslant 0$ be integers. Let h be a positive integer, where $h\neq 2$ or 6 if $m+2n<q-1$. Suppose there exist IMOLS$(h+v_i, v_i)$ for $1\leqslant i\leqslant m$ and IMOLS$(h+w_j, w_j)$ for $1\leqslant j\leqslant n$. Then there exists an HSOLS$(h^q(v+2w)^1)$, where $v=\Sigma v_i$ and $w=\Sigma w_j$.

Lemma 2.8 If(i) s is even and $s\notin\{2,6,10,14,46,54,58,62,66,70\}$; or (ii) s is an odd prime power exceeding 3, then there exists an HSOLS$(2^s(s-1)^1)$.

Proof In case (i), we use Theorem 2.6 and Lemma 2.7 with $q=s, m=s-1, n=0, h=2$ and $w_j=1$ we obtain an HSOLS $(2^s(s-1)^1)$.

In case (ii) we use Theorem 2.5 and the proof is similar.

3 MAIN RESULT

Lemma 3.1 There exist HSOLS$(2^n u^1)$ for $n=4,5,6,8$ and $2\leqslant u\leqslant n-1$.

Proof By Theorem1.1, HSOLS$(2^n u^1)$ for $n=4,5,6$ and $2\leqslant u\leqslant n-1$ and HSOLS$(2^8 u^1)$ for $2\leqslant u\leqslant 5$ are all exist.

HSOLS($2^8 6^1$) can be get by using Lemma 2. 1 with vectors $\theta=(\psi,15,3,6,9,11,13,X_1,\psi,14,x_2,x_3,x_4,x_5,x_6,10)$, $f=(12,1,5,7,4,2)$ and $g=(13,15,2,12,10,9)$.

HSOLS($2^8 7^1$) is from Lemma 2. 8 and the poof is complete.

Lemma 3. 2 If q is even and $q\notin\{2,6,10,14,46,54,58,62,66,70\}$, then there exist HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

Proof From Theorem 2. 6 we know that there exists an HSOLS(1^q) with q−1 ditjoint symmetric transversals. Applying Lemma 2. 7 with $m=q-1, n=0, h=8$ *and* $0\leqslant v_i\leqslant 4$, the input designs are from Theorem 2. 4, we obtain an HSOLS($8^q v^1$) for $0\leqslant v\leqslant 4(q-1)$. Further, applying Lemma 2. 2 with the input designs HSOLS ($2^4 k^1$)($2\leqslant k\leqslant 3$), we finally get an HSOLS($2^m u^1$) for $2\leqslant u\leqslant 4q-1$.

Lemma 3. 3 Let $n=mp, m\geqslant 4$ and (i) p is even and not in $\{2,6,10,14,46,54,58,62,66,70\}$, or (ii) $p\geqslant 5$ is an odd prime power. If there exist HSOLS($2^m k^1$) for $2\leqslant k\leqslant m-1$, then there exist HSOLS($2^n u^1$) for $2\leqslant u\leqslant n-1$.

Proof In case (i), applying Theorem 2. 6 and Lemma 2. 7 with $m=p-1, n=0, h=2m$ and $0\leqslant v_i\leqslant m$ we obtain an HSOLS($(2m)^p v^1$) for $0\leqslant v\leqslant m(p-1)$. Filling the holes of size $2m$ with HSOLS($2^m k^1$)($2\leqslant k\leqslant m-1$) we obtain an HSOLS($2^{2m}u^1$) for $2\leqslant u\leqslant mp-1$.

In case (ii) we apply Theorem 2. 5 and Lemma 2. 7 and the proof is similar.

Lemma 3. 4 There exist HSOLS($2^{4q}u^1$) for $q\in\{6,10,14,46,54,58,62,66,70\}$, and $2\leqslant u\leqslant 4q-1$.

Proof Applying Lemma 3. 3 with the following expressions of $n=4q=mp$ in Table 3. 1, the input designs are from Lemma 3. 1.

Table. 3. 1

q	6	10	14	46	54	8	62	66	70
$4q=mp$	6×4	5×8	8×7	8×23	8×27	8×29	8×31	6×44	(5×8)×7

Combine Lemmas 3. 1, 3. 2 and 3. 4 we have the following theorem.

Theorem 3. 5 If q is even, then there exist HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

Next, we consider the existence of HSOLS($2^{4q}u^1$) for q is odd.

Lemma 3. 6 There exist HSOLS($2^{12}u^1$) for $2\leqslant u\leqslant 11$.

Proof Applying Theorem 2. 5 and Lemma 2. 7 with $q=4, m=3, n=0, h=6$ and $1\leqslant v_i\leqslant 3$ we obtain an HSOLS($6^4 v^1$)for $3\leqslant v\leqslant 9$, Further, applying Lemma 2. 2 with HSOLS(2^4) we have an HSOLS($2^{12}u^1$) for $5\leqslant u\leqslant 11$.

HSOLS($2^{12}u^1$) for $2\leqslant u\leqslant 4$ are from Theorem 1. 1.

Lemma 3. 7 If q is an odd prime power exceeding 3, then there exist HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

Proof Applying Theorem 2. 5 and Lemma 2. 7 with $m=0, n=(q-1)/2, h=8$ and $0\leqslant w_j\leqslant 4$ we obtain an HSOLS ($8^q u^1$) for $0\leqslant v\leqslant 4(q-1)$, where v is even, Filling the holes of size eight with HSOLS($2^4 k^1$) ($2\leqslant k\leqslant 3$) we obtain an HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

Lemma 3. 8 If $q=3p, p$ is an odd prime power exceeding 3 and $3|p$, then there exist HSOLS($2^q u^1$) for $2\leqslant u\leqslant 4q-1$.

Proof Applying Theorem 2. 5 and Lemma 2. 7 with HSOLS(1^p), $m=0, n=(p-1)/2, h=24$ and $0\leqslant w_j\leqslant 12$, the input designs are from Theorem 2. 4, we obtain an HSOLS ($24^p v^1$) for $0\leqslant v\leqslant 12(p-1)$, where v is even Filling the holes of size 24 with HSOLS($24^p v^1$) ($2\leqslant k\leqslant 11$) we obtain an HSOLS($2^{12p}u^1$) for $2\leqslant u\leqslant 12p-1$. This completes the proof.

Appiying the Induction princeple with Lemmas 3. 1, 3. 5~3. 7 and Lemms 3. 3 we have the following theorem.

Theorem 3.9 If q is odd, then there exist HSOLS($2^{4q}u^1$) for $2\leqslant u\leqslant 4q-1$.

The main result of this paper becomes evident now.

Theorem 3.10 Suppose that $n\equiv 0 \pmod 4$, for $u\geqslant 2$, there exist HSOLS(2^nu^1) if and only if $n\geqslant 1+u$.

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