

### 惯性效应对绝热剪切失稳的影响

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# 惯性效应对绝热剪切失稳的影响

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摘 要:基于绝热剪切失稳和流体边界层的相似性,运用广义边界层理论方法,建立了绝热剪切失稳的控制方程,并结合量纲分析的方法对控制方程进行了无量纲化,得到了可以表征惯性效应的广义雷诺数,同时运用线性摄动的方法,对无量纲化后的控制方程组进行了求解,分析了惯性效应对绝热剪切失稳的影响.分析果构表明:当惯性效应达到一定临界值以后,绝热剪切失稳会被抑制,且这种抑制作用会随着惯性效应的增强而增强.

关键词:惯性效应;塑性稳定性;广义雷诺数;线性摄动理论;失稳相图 中图分类号:O344.7 文献标识码: A 开放科学(资源服务)标识码(OSID): 副 2

# Influence of Inertia Effect on Adiabatic Shear Instability LIU Jiejian<sup>1, 2</sup>, WU Bo<sup>2</sup>, YAN Xiaojing<sup>2</sup>, WANG Genwei<sup>3\*</sup>

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**Abstract**: Based on the similarity between adiabatic shear instability and fluid boundary layer, the governing equations of adiabatic shear instability are established by the generalized boundary layer theory. The governing equations are dimensionless by the method of dimensional analysis, and the generalized Reynolds number which can represent the inertial effect is obtained. At the same time, the dimensionless governing equations are solved by the method of linear perturbation. The influence of inertia effect on adiabatic shear instability is analyzed. The results showed that the adiabatic shear instability would be restrained when the inertia effect reaches a certain critical value, and the restraining effect would increase with the increase of inertia effect.

Key words: inertia effect; plastic stability; generalized Reynold number; linear perturbation; phase diagram

#### 0 引言

ZENER 和 HOLLOMON<sup>[1]</sup>最早研究了金属 材料绝热剪切失稳问题,首次通过试验分析得到, 当应力应变曲线的斜率为负值时,塑性变形已经从 均匀变形转变成局部集中的塑性变形.且这种负的 斜率是由两种机制相互竞争决定的,即加工硬化效 应和热软化效应.当加工硬化的作用大于热软化的 作用时,此时塑性变形是稳定的,即变形是均匀的. 反之,则塑性变形会失去稳定性,就会形成绝热剪 切失稳.随后,BAI<sup>[2]</sup>运用线性摄动近似求解的方 法,建立简单剪切加载条件下(一维剪切变形)的绝 热剪切失稳的判据. BAI 的判据从理论上量化了 加工硬化效应、应变率硬化效应和热软化效应在绝 热剪切失稳中的作用.当硬化的作用小于软化的作 用的时候,系统就会发生塑性失稳.MA 和LIU<sup>[3-4]</sup> 研究了在复杂应力加载条件下发生塑性失稳的条 件,这些塑性失稳包括颈缩塑性失稳、绝热剪切塑 性失稳,同时结合塑性失稳相图来判断加工硬化效 应、应变率效应和热软化效应在塑性失稳中的作 用,由此建立了二维复杂变形条件下的统一塑性失 稳判据.MOLINARI<sup>[5]</sup>通过数值模拟的方法,研究

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了加工硬化效应、应变率硬化效应、热软化效应、惯 性效应对金属材料绝热剪切失稳的影响.但是,到 目前为止,还没有文献报道惯性效应在塑性失稳中 作用的理论研究.

本文基于广义边界层<sup>[6]</sup>理论,运用 Navier-Stokes 方程、连续性方程、应变演化方程、变形协 调方程和能量方程建立可以描述加工硬化效应、应 变率硬化效应、热软化效应、惯性效应对金属材料 绝热剪切失稳影响的理论模型.并运用线性摄动理 论对此理论模型进行了近似求解,结合塑性失稳相 图研究了惯性效应对绝热剪切失稳的影响.

#### 1 理论模型的建立

金属材料在高应变率的加载条件下,通常会形成一条窄的剧烈塑性变化区域(见图 1).这种剪切带与流体力学中的边界层相似,是一个很薄的、具有大的变形梯度的区域,其厚度的量级通常在10~ 100 μm<sup>[7]</sup>,同时伴有局部温度的剧烈变化<sup>[8+9]</sup>,并且带的传播速度很快,有时能超过 1000 ms<sup>-1[10-11]</sup>.这种问题的建模通常需要考虑塑形大变形、率敏感性、加工硬化、热传导、热软化和惯性效应等因素.纯粹通过固体力学中定理方程已经很 难将这些影响因素考虑进去,必须适当借助流体力学边界层的相关理论进行研究.





**Fig.** 1 Schematic diagram of adiabatic shear instability N-S 方程具有如式(1)的数学表达形式:

$$\begin{cases} \rho(\frac{\partial v_1}{\partial t} + v_1 \frac{\partial v_1}{\partial x_1} + v_2 \frac{\partial v_1}{\partial x_2}) = \\ \frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{12}}{\partial x_2} + \frac{\partial p}{\partial x_1}, \\ \rho(\frac{\partial v_2}{\partial t} + v_1 \frac{\partial v_2}{\partial x_1} + v_2 \frac{\partial v_2}{\partial x_2}) = \\ \frac{\partial \tau_{21}}{\partial x_1} + \frac{\partial \tau_{22}}{\partial x_2} + \frac{\partial p}{\partial x_2}, \end{cases}$$
(1)

式中: $\rho$  表示材料的密度, $v_1$ 、 $v_2$ 表示欧拉速度场,p为静水压力(正压力), $\tau_{ij}(i,j=1,2)$ 表示应力偏 量;t 为时间变量; $x_1$  代表 x 方向; $x_2$  代表 y 方向. 在高速动态加载条件下,假定材料的弹性变形相对 塑形变形可以忽略不计,且认为材料密度不变(不可压缩性),则二维条件下的连续性方程为:

$$\frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} = 0.$$
 (2)

同时,材料的变形必须满足变形协调条件:

$$\frac{\partial^2 \boldsymbol{\varepsilon}_{11}}{\partial x_2^2} + \frac{\partial^2 \boldsymbol{\varepsilon}_{22}}{\partial x_1^2} = 2 \frac{\partial^2 \boldsymbol{\varepsilon}_{12}}{\partial x_1 \partial x_2}, \qquad (3)$$

其中:ε<sub>ij</sub>(*i*,*j*=1,2)为应变场应变值的大小,由于 是在欧拉场研究材料的稳定性,则应变随时间的演 化方程为:

$$\dot{\boldsymbol{\varepsilon}}_{ij} = \frac{\partial \boldsymbol{\varepsilon}_{ij}}{\partial t} + \boldsymbol{v}_1 \, \frac{\partial \boldsymbol{\varepsilon}_{ij}}{\partial x_1} + \boldsymbol{v}_2 \, \frac{\partial \boldsymbol{\varepsilon}_{ij}}{\partial x_2}, (i, j = 1, 2),$$
(4)

温度的分布则由能量方程控制:

$$\rho C_{P} \left( \frac{\partial T}{\partial t} + v_{1} \frac{\partial T}{\partial x_{1}} + v_{2} \frac{\partial T}{\partial x_{2}} \right) = K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right) + K \left( \frac{\partial^{2} T}{\partial x_{1}^{2}} + \frac{\partial^{2} T}{\partial x_{2}^{2}} \right)$$

 $\beta(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + 2\sigma_{12}\varepsilon_{12}), \quad (5)$ 其中: $C_P$ 为比定压热容:K为热传导系数; $\beta$ 为泰

具中:C<sub>P</sub>为比定压热谷;K 为热传导系数;β 为泰 勒-奎尼系数,用来表征功热转化的百分率,其数值 大小一般取 0.90~0.95.

#### 2 控制方程的无量纲化

采用广义边界层理论研究材料的稳定性,运用 流体力学中的无量纲化方法:

$$\hat{x}_{1} = \frac{x_{1}}{L_{0}}, \hat{x}_{2} = \frac{x_{2}}{\varepsilon L_{0}}, \hat{v}_{i} = \frac{v_{i}}{U_{0}}, \hat{t} = \frac{U_{0}t}{L_{0}},$$
$$\hat{\sigma}_{ij} = \frac{L_{0}\varepsilon_{0}\sigma_{ij}}{\sigma_{0}U_{0}}, \hat{T} = \frac{T}{T_{m}}, \qquad (6)$$

式中:L<sub>0</sub>为材料在流动方向的特征长度, *ε*为小量, 通过 *εL*<sub>0</sub>表征垂直于流动方向的特征长度;*U*<sub>0</sub>为外 力的加载速度; *σ*<sub>0</sub>为材料的初始屈服应力; *T*<sub>m</sub>为材 料的熔点.完成无量纲化后,可以得到广义雷诺数 *Re*,表征惯性效应在绝热剪切失稳中的作用.

#### 3 控制方程的摄动分析

在进行线性摄动分析的时候,采用式(7)中所示的通用描述形式:

$$\{ \stackrel{\wedge}{\varphi}_{ij} \} = \{ \stackrel{\wedge}{\varphi}_{ij0} \} + \{ \stackrel{\wedge}{\delta \varphi}_{ij} \} e^{\stackrel{\wedge}{\omega} \stackrel{\wedge}{t} + i(\stackrel{\wedge}{k}_{1} \stackrel{\wedge}{x}_{1} + \stackrel{\wedge}{k}_{2} \stackrel{\wedge}{x}_{2})},$$

$$(i, j = 1, 2),$$

$$(7)$$

其中:  $\{\hat{\varphi}_{ij0}\} = \{\hat{v}_{i0}, \hat{\sigma}_{ij0}, \hat{\epsilon}_{ij0}, \hat{\epsilon}_{ij0}, \hat{T}_{0}\}$ ,且满足  $\{\hat{\varphi}_{ij0}\} \gg \{\hat{\delta}\hat{\varphi}_{ij}\}; \hat{v}_{i0}, \hat{\sigma}_{ij0}, \hat{\epsilon}_{ij0}, \hat{\epsilon}_{ij0}, \hat{T}_{0}\}$ 为招 制方程组的稳定解;而  $\delta_{v_{i0}}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、 $\delta_{c_{ij0}}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、 $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$  $\delta_{ij0}^{\circ}$ 、  $\delta_{ij0}^{\circ}$  $\delta_$ 

$$\begin{cases} -C_{10} \hat{P}_{0} \hat{k}_{0}^{2} \delta \hat{T} + (C_{10} C_{11} \hat{Q}_{0} \hat{k}_{0}^{2} + \\ C_{10} C_{11} \hat{R}_{0} \hat{k}_{0}^{2} \hat{\omega} ) \delta \hat{\varepsilon}_{N} + (2C_{10} \hat{Q}_{0} \hat{k}_{0}^{2} + \\ 2C_{10} \hat{R}_{0} \hat{k}_{0}^{2} \hat{\omega} + 2Re^{2} \hat{\omega}^{2} ) \delta \hat{\varepsilon}_{12} + \\ C_{12} \delta \hat{v}_{s} = 0 \\ - (C_{13} \Lambda \hat{k}_{0}^{2} + C_{14} \Phi \hat{P}_{0} + \hat{\omega}) \delta \hat{T} + \\ (\Phi(\eta_{11} \hat{\sigma}_{11(0)} + \eta_{22} \hat{\sigma}_{22(0)}) \hat{\omega} + \\ C_{11} C_{14} \Phi \hat{R}_{0} \hat{\omega} + C_{11} C_{14} \Phi \hat{Q}_{0}) \delta \hat{\varepsilon}_{N} + \\ (2\Phi Re \hat{\sigma}_{12(0)} \hat{\omega} + 2C_{14} \Phi \hat{R}_{0} \hat{\omega} + \\ 2C_{14} \Phi \hat{Q}_{0}) \delta \hat{\varepsilon}_{12} + C_{15} \delta \hat{v}_{s} = 0, \\ C_{16} \delta \hat{\varepsilon}_{N} - C_{17} \delta \hat{\varepsilon}_{12} = 0, \\ C_{18} \delta \hat{v}_{s} = 0, \end{cases}$$

(8)

式中:Q 为应变硬化系数;η 为各应变分量在总应 变中所占的比重,由数值模拟结果给出;Φ 为无量 纲的热传导系数;Λ 为无量纲的功热转换系数.要 使齐次方程组(8)有解,则必须保证方程组(8)的四 阶系数行列式的值为零,由此可得:

$$2C_{23}Re^{2}\hat{\omega}^{3} + (2C_{16}C_{23}Re^{2}\Phi\hat{P}_{0} + 2C_{17}C_{23}Re^{2}\Lambda\hat{k}_{0}^{2} + (C_{14}C_{23} + C_{12}C_{24})\hat{R}_{0}\hat{k}_{0}^{2}\hat{k}_{0}^{2})\hat{\omega}^{2} + ((C_{14}C_{17}C_{23} + C_{12}C_{17}C_{24})\Lambda\hat{R}_{0}\hat{k}_{0}^{4} + (C_{14}C_{16}C_{23} + C_{12}C_{16}C_{24} - C_{10}C_{18}C_{24} - C_{10}C_{20}C_{23})\Phi\hat{P}_{0}\hat{R}_{0}\hat{k}_{0}^{2} - C_{10}C_{24}(\eta_{11}\hat{\sigma}_{11} + \eta_{22}\hat{\sigma}_{22})\Phi\hat{P}_{0}\hat{k}_{0}^{2} - 2C_{10}C_{23}\Phi\hat{\sigma}_{12}\hat{P}_{0}\hat{k}_{0}^{2} + (C_{13}C_{17}C_{23} + C_{11}C_{12})\hat{Q}_{0}\hat{k}_{0}^{2})\hat{\omega} + (C_{13}C_{17}C_{23} + C_{11}C_{17}C_{24})\Lambda\hat{Q}_{0}\hat{k}_{0}^{4} + (C_{13}C_{16}C_{23} + C_{11}C_{16}C_{24} - C_{10}C_{19}C_{24} - C_{10}C_{21}C_{23})\Phi\hat{P}_{0}\hat{Q}_{0}\hat{k}_{0}^{2} = 0.$$
(9)

显然式(9)是一个扰动增长率的三次方程,想要直

接求解是比较困难的,采用以波数 k<sup>2</sup> 作为未知量 来重组方程,最后达到求解系统稳定性的判据.求 解过程如下:

$$a \dot{k}_{0}^{4} + b \dot{k}_{0}^{2} + c = 0, \qquad (10)$$

由式(10)可知,重组后的方程组为波数 k<sup>2</sup> 作为未 知量的一元二次方程组,利用一元二次方程组的求 根公式可得:

$$\hat{k}_{0}^{2} = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a},$$
(11)

对方程(10)两边同时对  $\hat{k}_{0}^{2}$  求导,即 $\frac{\partial \hat{\omega}}{\partial k_{0}^{2}}=0$ 可得:

$$\hat{k}_{0}^{2} = \frac{-b}{2a}.$$
(12)

比较式(11)与式(12)可得: $b^2 - 4ac = 0$ .令 $G_1(\overset{\land}{\omega})$ = $b^2 和 G_2(\overset{\land}{\omega}) = 4ac$ 可得(对于绝热剪切,热传导 效应可以忽略不计),

$$G_{1}(\stackrel{\frown}{\omega}) = ((2C_{17}C_{23}Re^{2} + (C_{14}C_{23} + C_{12}C_{24})\stackrel{\frown}{R}_{0}/\Lambda)\stackrel{\frown}{\omega} + ((C_{14}C_{16}C_{23} + C_{12}C_{16}C_{24} - C_{10}C_{18}C_{24} - C_{10}C_{20}C_{23})\Phi\stackrel{\frown}{P}_{0}\stackrel{\frown}{R}_{0}(\rho U^{2}/Q_{0}) - C_{10}C_{24}(\eta_{11}\stackrel{\frown}{\sigma}_{11} + \eta_{22}\stackrel{\frown}{\sigma}_{22})\Phi\stackrel{\frown}{P}_{0}(\rho U^{2}/Q_{0}) - 2C_{10}C_{23}\Phi\stackrel{\frown}{\sigma}_{12}\stackrel{\frown}{P}_{0}(\rho U^{2}/Q_{0}) + (C_{13}C_{23} + C_{11}C_{24})\stackrel{\frown}{Q}_{0}(\rho U^{2}/Q_{0}))^{2},$$
(13)  
$$G_{2}(\stackrel{\frown}{\omega}) = 8C_{23}Re^{2}\stackrel{\frown}{\omega}((C_{14}C_{17}C_{23} + C_{12}C_{17}C_{24})(\stackrel{\frown}{R}_{0}/\Lambda)\stackrel{\frown}{\omega} + (C_{13}C_{17}C_{23} + C_{11}C_{17}C_{24})\stackrel{\frown}{Q}_{0}(\rho U^{2}/Q_{0})).$$
(14)

要得到扰动增长率 $\omega$ 的正根,必须满足 $G_1(0) > G_2(0)$ ,可得:

$$\Phi \stackrel{\wedge}{P}_{0} (C_{10}C_{24}(\eta_{11}\stackrel{\wedge}{\sigma}_{11}+\eta_{22}\stackrel{\wedge}{\sigma}_{22})+2C_{10}C_{23}\stackrel{\wedge}{\sigma}_{12}- (C_{14}C_{16}C_{23}+C_{12}C_{16}C_{24}-C_{10}C_{18}C_{24}- C_{10}C_{20}C_{23}\stackrel{\wedge}{R}_{0}) > (C_{13}C_{23}+C_{11}C_{24})\stackrel{\wedge}{Q}_{0}.$$
(15)

#### 4 绝热剪切失稳相图分析

式(15)已经得到了绝热剪切失稳的理论判据, 本节通过数值模拟和绝热剪切失稳相图来说明惯 性效应对绝热剪切失稳的影响.金属切削过程中发 生的塑性失稳作为一种典型的绝热剪切失稳模式, 本节将通过金属的切削失稳,结合第3节得到失稳 判据来说明惯性效应对绝热剪切失稳的影响.数值 模拟中采用 Johnson-Cook 本构关系模型,数值模 拟中采用的材料参数可以参见文献[5].在金属的 切削速度分别为 50、70、100、150、170、190、210、 250、290 m/s时,临界控制函数(13)和(14)在不同 切削速度下,扰动增长率与切削速度的变化关系曲 线如图 2 所示.



图 2 切削速度与扰动增长率的变化关系曲线 Fig. 2 The relation between cutting speed and perturbation growth rate

由图 2 可知,在低速切削阶段,扰动增长率随着切削速度的增大而增大;当切削速度增大到一定 程度时,扰动增长率随着切削速度的增大而减小. 这和由数值模拟所得的切削力的波动与切削速度 的变化趋势一致.

#### 5 结论

在低应变率的加载条件下,惯性效应在绝热剪 切失稳中作用很小,扰动增长率的数值大小为零 值,材料不会发生绝热剪切失稳;随着应变率的增 大,惯性效应的作用不断增强,但是此时热软化还 是主控因素,此时材料会发生绝热剪切失稳;当应 变率超过一定的数值,惯性效应成为主控因素,扰 动增长率的数值开始不断减小,随着惯性效应的继 续增强,扰动增长率将趋向于零,此时将不易发生 绝热剪切失稳.本文通过理论研究,揭示了金属材 料中惯性效应对绝热剪切失稳的影响,但是不同材 料的物理性质和本构关系不相同,因此对于具体的 金属材料,需要结合实验和本构关系进一步研究.

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